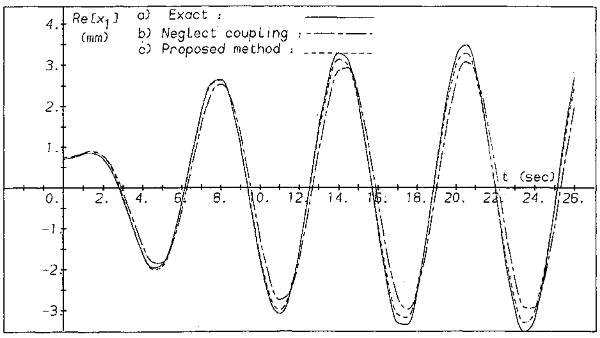

Fig. 3 Real part of the response at DOF-1 vs time. Excitation with frequency: $\omega_s = 1.0$ rad/s.

The eigenproblem of the undamped system yields natural frequencies $\omega_1 = 1.0$, $\omega_2 = 1.1$, and $\omega_3 = 3.0$ (ω_1 and ω_2 closely spaced), and modal matrix

$$\Psi = \begin{bmatrix} 0.707 & -0.707 & 0.000 \\ 0.707 & 0.707 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

Note that $DM^{-1}K \neq KM^{-1}D$ and, therefore, the system under study does not possess normal modes, i.e., it is not diagonalizable. In order to form the fictitious damping ratios, we applied the formula

$$\xi_{is} = \frac{\sum_{j=1}^{3} \tilde{d}_{ij}}{2(\omega_i \omega_s)^{1/2}}, \qquad i = 1,2,3$$

which leads to the classical definition of the damping ratios for a proportionally damped system, i.e., for $\tilde{d}_{ij} = 0$ $(i \neq j)$ and $\omega_s = \omega_i$. Notice here that the proposed method is capable of handing problems with frequency-dependent damping ratios as, for example, in several applications of viscoelastic materials with frequency-dependent damping properties.

In Fig. 1 we present the frequency spectrum of nonproportionality indices. By inspection one can easily realize where the modal coupling becomes an important factor, and when the off-diagonal terms of the transformed damping matrix $\tilde{D}$ are negligible.

From the coordinate transformation [Eq. (2)] and the form of the previously given modal matrix Ψ, we conclude that only the first two modes participate in each of the first two degrees of freedom, and only the third mode in the third degree of freedom. To be more concise and maintain the physical sense, we illustrate next only the real part of the response. In Fig. 2 we present the response of the first degree of freedom for excitation frequency $\omega_s = \omega_1 = 1.0$ rad/s. As the nonproportionality indices predict (see Fig. 1), the highest deviations occur when one ignores the coupling in the dominant second mode. In fact, it is clear that we can neglect the coupling in the first mode throughout the frequency range without introducing significant errors.

Figure 3 was formed to compare the solution given by our method, to the exact solution as well as to the one obtained by neglecting the coupling completely. Note that our proposed approximate method results in steady-state responses close to the exact ones, and corrects those responses resulting by simply neglecting the off-diagonal terms of the transformed damping matrix $\tilde{D}$, where the consideration of the modal coupling becomes important. Deviations from the exact solution should occur in the transient part of the response, due to the decoupling procedure we used. The primary advantage of the proposed method is the computational simplicity (recalling that the state formulation of the problem requires matrices and vectors belonging to a $2n$-dimensional space).

Acknowledgment

The authors acknowledge the support of the Mechanical and Aerospace Engineering Department and the Mechanical Systems Laboratory of the State University of New York at Buffalo, the National Science Foundation (Grant MSM 8351807), and the Air Force Office of Scientific Research (Grants AFOSR -850220 and AFOSR F 49620-86-C-011).

References

[1]Seireg, A. and Howard, L., "An Approximate Normal Mode Method for Damped Lumped Parameter Systems," *Journal of Engineering for Industry*, Vol. 2, Nov. 1967, pp. 597–604.

[2]Warburton, G. B. and Soni, S. R., "Errors in Response Calculations for Non-Classically Damped Structures," *Earthquake Engineering and Structural Dynamics*, Vol. 5, 1977, pp. 365–376.

[3]Duncan, P. E. and Eatock, T. R., "A Note on the Dynamic Analysis of Non-Proportionally Damped Systems," *Earthquake Engineering and Structural Dynamics*, Vol. 7, 1979, pp. 99–105.

[4]Özgüven, H. N., "Modal Analysis of Non-Proportionally Damped Mechanical Systems," American Society of Mechanical Engineers, Paper 81-DET-75, 1981.

[5]Singh, R., Prater, G., Jr., and Nair, S., "Quantification of the Extent of Non-Proportional Viscous Damping in Discrete Vibratory Systems," *Journal of Sound and Vibration*, Vol. 104, 1986, pp. 109–125.

[6]Singh, R. and Nair, S., "Examination of the Validity of Proportional Damping Approximations," *Journal of Sound and Vibration*, Vol. 104, 1986, pp. 348–350.

[7]Caughey, T. K. and O'Kelly, M. E. J., "Classical Normal Modes in Damped Linear Dynamic Systems," *ASME Journal of Applied Mechanics*, Vol. 32, Sept. 1965, pp. 583–588.

[8]Melsa, J. L. and Schultz, D. G., *Linear Control Systems*, McGraw-Hill, New York, 1969, p. 187.

[9]Bellos, J., "Vibrations of Non Proportionally Damped Structures," M.S.Thesis, State Univ. of New York at Buffalo, 1987, pp. 106–111.

Bounded-Input/Bounded-Output Stability of Linear Multidimensional Time-Varying Systems

S. Pradeep* and S. K. Shrivastava†
Indian Institute of Science, Bangalore, India

Introduction

In recent years, the attention of several authors has been attracted by the stability of systems governed by the equation

$$M(t)q''(t) + G(t)q'(t) + K(t)q(t) = B(t)v(t) \qquad (1)$$

where primes denote differentiation with respect to time t, the independent variable; q is an $(n \times 1)$ vector; M, G, K are $(n \times n)$ real matrices (whose elements are functions of t and differentiable); B is an $(n \times m)$ real matrix (whose elements are functions of t); v is an $(m \times 1)$ vector; and n is the dimension of the system (which is often large).

Such systems are encountered in spacecraft dynamics, economics, ecology, biosystems, demography, and several engineering disciplines. With the exception of trivial cases, an explicit solution of Eq. (1) is impossible. The only way to

Received July 20, 1987; revision received June 13, 1988.

*Research Associate, Department of Aerospace Engineering.

†Professor, Department of Aerospace Engineering. Associate Fellow, AIAA.

comprehend the behavior of the system is to study the properties of its solution. Of all properties of the solution, stability is perhaps the most fundamental. Scientific literature abounds in studies in stability.[1] However, most of the available stability criteria are meant for first-order systems of the form

$$x'(t) = B(t)x(t) + u(t) \tag{2}$$

To apply the existing criteria, Eq. (1) must be converted to this first-order form. This may be done in the following way:

Let

$$x(t) = [\boldsymbol{q}'(t)\ \boldsymbol{q}(t)]^T \tag{3a}$$

and

$$U(t) = [B(t)\boldsymbol{v}(t)\ 0]^T \tag{3b}$$

Then,

$$x'(t) = \begin{bmatrix} -M^{-1}G & -M^{-1}K \\ I & 0 \end{bmatrix} x(t) + U(t) \tag{3c}$$

This method of conversion requires the inversion of M and subsequent multiplication with G/K. These two processes, in addition to being laborious (owing to the large dimension of the system), are also computationally and analytically unwieldy. These difficulties may be surmounted by treating Eq. (1) as it is, or by employing alternative transformations that do not require inversion and multiplication of matrices. One such transformation would be: with x and U, as defined in Eq (3),

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} x' = \begin{bmatrix} -G & -K \\ I & 0 \end{bmatrix} x + U \tag{4}$$

It is seen that although Eq. (4) is still of the first order, it is a shade different from Eq. (2). Explicitly, it is of the form

$$A(t)x'(t) = B(t)x(t) + U(t) \tag{5}$$

The authors have been involved in devising new methods[2-5] of stability analysis for systems governed by Eqs. (1) and (5). In this Note, L_∞ stability of systems of the form of Eq. (5) is analyzed, using the methods of functional analysis.

Analysis

Functional analytic methods deal with a pair of implicit equations: $u_1 = e_1 + H_2 e_2$ and $u_2 = e_2 - H_1 e_1$, where u_1, u_2, e_1, e_2 belong to some extended functional space X_e, while H_1 and H_2 map X_e into itself. If the input u_2 is zero, the preceding equations reduce to

$$u_1 = e_1 + H_2 H_1 e_1 \tag{6}$$

As mentioned earlier, most of the available input-output results deal with first-order systems of the form of Eq. (2). Our intent is to examine equations of the form of Eq. (1), or, at least of Eq. (5). By integrating twice with respect to t, Eq. (1) can be converted into an integral equation of the same form as Eq. (6). However, this equation is difficult to analyze, owing to the arbitrarily varying operators it contains. To simplify analysis, the fact that all linear time-invariant operators that satisfy some slight continuity properties have a convolutional representation, is often used. Thus, in most of the work on input-output stability, the arbitrarily varying system is split into a linear time-invariant system in the forward path (represented by H_1) and a time-varying linear or nonlinear system in the feedback path (represented by H_2). Adopting the same line of thought leads to a system of the form

$$[P + A(t)]\boldsymbol{q}'' + [Q + B(t)]\boldsymbol{q}' + [R + C(t)]\boldsymbol{q} = u(t) \tag{7}$$

which is equivalent to the integro-differential equation

$$\boldsymbol{q}(t) = \int_0^t k(t-\tau)\{u - A(\tau)(\mathrm{d}^2\boldsymbol{q}/\mathrm{d}\tau^2) - B(\tau)(\mathrm{d}\boldsymbol{q}/\mathrm{d}\tau) - C(\tau)\boldsymbol{q}\}\,\mathrm{d}\tau + \boldsymbol{v}(t) \tag{8}$$

where $P\boldsymbol{v}'' + Q\boldsymbol{v}' + R\boldsymbol{v} = 0$ and $k(t) = \mathscr{L}^{-1}\{(s^2P + sQ + R)^{-1}\}$.

This integro-differential equation may be converted into an integral equation in $\boldsymbol{q}(t)$ by integrating the $\mathrm{d}\boldsymbol{q}/\mathrm{d}t$ and the $\mathrm{d}^2\boldsymbol{q}/\mathrm{d}t^2$ terms by parts. Further analysis requires a closed-form expression for $k(t)$. It is well known that when P, Q, and R are scalars, such a closed-form expression exists and is in terms of the product of a decaying exponential and a sine function. Unfortunately, such a solution does not carry over to the case in which P, Q, and R are matrices; in this case, it has not been possible to find a closed-form solution for $k(t)$. However, on converting Eq. (7) to the first-order form

$$[I + C(t)]\boldsymbol{x}'(t) = [B + D(t)]\boldsymbol{x}(t) + u(t) \tag{9}$$

it is observed that the following closed-form expression exists for the corresponding convolution operator $k(t)$:

$$k(t) = \mathscr{L}^{-1}\{(sI - B)^{-1}\} = e^{Bt} \tag{10}$$

Therefore, instead of the general form of Eq. (7), the first-order form of Eq. (9) has been analyzed in the sequel.

A lemma is first proved, and using it, a theorem on L_∞ stability is proved. The lemma is a modification of Sandberg's result[6] to suit the special system on hand.

Lemma. Let $k(t)$, $Q(t)$, and $C(t)$ denote measurable $(n \times n)$ matrix-valued functions of t defined on $[0,\infty)$. Let $k(t)$ possess elements $k_{ij}(t)$, such that for $p = 1,2$,

$$\int_0^\infty |e^{c_1 t} k_{ij}(t)|p\ \mathrm{d}t < \infty, \qquad i,j = 1,2,\ldots,n \tag{11}$$

for some real constant c_1, and let the elements of $Q(t)$ and $C(t)$ be uniformly bounded on $[0,\infty)$. Let g and f denote measurable n-vector valued functions of t defined on $[0,\infty)$, such that

$$ge^{c_1 t} \in L_2^n[0,\infty), \qquad ge^{c_1 t} \to 0 \text{ as } t \to \infty$$

$$g(t) = [I + C(t)]f(t) - \int_0^t k(t-\tau)Q(\tau)f(\tau)\,\mathrm{d}\tau, \quad t \geq 0 \tag{12}$$

Suppose that with

$$K_w(i\omega) \triangleq \int_0^\infty k(t)e^{(c_1 - i\omega)t}\,\mathrm{d}t, \qquad \|C(t)\| \leq c < 1,$$

$$r \triangleq \sup_{t\geq 0} \Lambda\{Q(t)\} \sup_{\omega\in\mathbb{R}} \Lambda\{K_w(i\omega)\}, \qquad c + r < 1 \tag{13}$$

then $f \in L_\infty^n[0,\infty)$, and

$$\|f(t)\| \leq [1/(1-c)][\|g(t)\| + c_2 e^{-c_1 t}] \tag{14}$$

Proof. On multiplying both sides of Eq. (12) by $e^{c_1 t}$,

$$e^{c_1 t}g(t) = [I + C(t)]\tilde{f}(t) - \int_0^t e^{c_1(t-\tau)}k(t-\tau)Q(\tau)\tilde{f}(\tau)\,\mathrm{d}\tau \tag{15}$$

where $\tilde{f}(t) = e^{c_1 t}f(t)$.

Using a result of Ref. 5, with the definitions as given in the statement of the lemma and with the relations given by Eq. (13), $\tilde{f} \in L_2^n[0,\infty)$. Thus, $e^{c_1 t}Q(t)f(t) \in L_2^n[0,\infty)$ and $f(t) \in L_2^n[0,\infty)$.

By the Schwarz inequality, there exists a positive constant c_2, such that

$$\int_0^t e^{c_1(t-\tau)}k(t-\tau)e^{c_1\tau}Q(\tau)f(\tau)\,\mathrm{d}\tau \leq c_2, \qquad t \geq 0 \tag{16}$$

Thus, using Eq. (15),

$$\|f(t)\| \le \|C(t)\| \cdot \|f(t)\| + \|g(t)\| + c_2 e^{-c_1 t}$$
$$\le c\|f(t)\| + \|g(t)\| + c_2 e^{-c_1 t}$$
$$\|f(t)\| \le 1/(1-c)[\|g(t)\| + c_2 e^{-c_1 t}] \tag{17}$$

The main result is now presented.

Theorem. Let B denote a constant $(n \times n)$ matrix and c_1 denote a positive real number, such that all the eigenvalues of $(B + c_1 I)$ have negative real parts. Let $C(t)$ and $D(t)$ denote $(n \times n)$ matrix functions of t, with measurable and uniformly bounded elements for $t \ge 0$. (The elements of C are assumed to be differentiable.) Let $\boldsymbol{x}$ be a complex n-vector valued, differentiable function of t on $[0,\infty)$, which satisfies Eq. (9), with $e^{c_1 t}u(t) \in L_2^n[0,\infty)$ for $c_1 > 0$.

If $(\xi_j + i\eta_j), j = 1,...,n$ are the eigenvalues of B, then let $\xi \triangleq \{\xi_j: |\xi_j - c_1|$ is minimum over $j = 1,...,n\}$.

If $\|C(t)\| \le c < 1$, and

$$c + 1/|\xi - c_1| \sup_{t\ge 0} \Lambda[D(t) + C'(t) - BC(t)] < 1 \tag{18}$$

then there exists c_2, c_3, c_4, all positive, such that

$$x \in L_\infty^n[0,\infty)$$

and

$$\|x(t)\| \le 1/(1-c)[c_2 e^{-c_1 t} + c_3 + c_4 \int_0^t \|u(\tau)\| \,\mathrm{d}\tau] \qquad \text{for } t \ge 0 \tag{19}$$

Proof. Equation (9) is equivalent to the integral equation

$$g(t) = [I + C(t)]x(t) - \int_0^t k(t-\tau)\{D(\tau) + [\mathrm{d}C(\tau)/\mathrm{d}\tau] - BC(\tau)\}x(\tau)\,\mathrm{d}\tau \tag{20}$$

where

$$g(t) = v(t) + w(t) + k(t)C(0)x(0), \qquad Iv' = Bv,$$
$$w(t) \triangleq \int_0^t k(t-\tau)u(\tau)\,\mathrm{d}\tau, \qquad k(t) = \mathscr{L}^{-1}\{(sI - B)^{-1}\} = e^{Bt} \tag{21}$$

Now, $\boldsymbol{v}(t) = e^{Bt}$ and $e^{c_1 t}e^{Bt} \in L_2^n[0,\infty)$ by assumption. Likewise, $e^{c_1 t}w(t)$ and $e^{c_1 t}k(t)C(0)x(0)$ also belong to $L_2^n[0,\infty)$. Hence, $e^{c_1 t}g(t) \in L_2^n[0,\infty)$.

Applying the lemma to Eq. (20), if $\|C(t)\| \le c < 1$, and if

$$c + \sup_{t\ge 0} \Lambda\{Q(t)\} \sup_{\omega\in\mathbb{R}} \Lambda[K_w(i\omega)] < 1 \tag{22}$$

then

$$x \in L_\infty^n[0,\infty) \quad \text{and} \quad \|x(t)\| \le 1/(1-c)(\|g(t)\| + c_2 e^{-c_1 t}) \tag{23}$$

Now,

$$K_w(i\omega) = \int_0^\infty e^{Bt}e^{(c_1 - i\omega)t}\,\mathrm{d}t = -(B + c_1 I - i\omega I)^{-1} \tag{24}$$

The eigenvalues of $(B + c_1 I - i\omega I)^{-1}$ are the same as those of $(\beta + c_1 I - i\omega I)^{-1}$, where $\beta = \text{diag}$ (eigenvalues of B). Therefore,

$$\Lambda\{(B + c_1 I - i\omega I)^{-1}\} = \Lambda\{(\beta + c_1 I - i\omega I)^{-1}\} \tag{25}$$

where $\Lambda(A)$ = positive square root of the largest eigenvalue of A^*A. It may be shown that $\sup_{\omega\in\mathbb{R}}\Lambda[K(i\omega)] = (1/|\xi - c_1|)$. Hence,

$$c + (1/|\xi - c_1|) \sup_{t\ge 0} \Lambda\{D(t) + C'(t) - BC(t)\} < 1 \tag{26}$$

is sufficient for $x \in L_\infty^n\ [0,\infty)$. Now,

$$g(t) = v(t) + w(t) + k(t)C(0)x(0) \tag{27}$$

and hence,

$$\|g(t)\| \le \|v(t)\| + \|w(t)\| + \|k(t)\| \cdot \|C(0)\|$$
$$\le k_1 + \left\| \int_0^t k(t-\tau)u(\tau)\,\mathrm{d}\tau \right\| + k_2 \tag{28}$$

for positive constants k_1 and k_2

$$\le k_1 + k_2 + \int_0^t c_4\|u(\tau)\|\,\mathrm{d}\tau, \qquad \text{for } c_4 > 0 \tag{29}$$

With the definition $c_3 \triangleq k_1 + k_2$, Eq. (23) reduces to

$$\|x(t)\| \le 1/(1-c)[c_2 e^{-c_1 t} + c_3 + c_4 \int_0^t \|u(\tau)\|\,\mathrm{d}\tau] \tag{30}$$

Remarks. 1) Although the mathematical analysis may seem a little abstract, the theorem is quite easy to apply to most engineering systems. The equations governing even the most complex systems are well behaved mathematically; the elements of the matrices C and D are almost invariably measurable, uniformly bounded, and differentiable. It is easy to verify that all the eigenvalues of the constant matrix B have negative real parts. Verification of Eq. (18) may be simplified by using inequality

$$\Lambda\{A\} \le n \max_{j,k}|a_{jk}| \tag{31}$$

2) It must be borne in mind that the input-output methods based on functional analysis only provide sufficient conditions for stability that may not exactly duplicate the actual stability boundaries. (This can be verified for a specific system such as the Mathieu equation.) Nevertheless, these results are important, for while analyzing a system, it is advantageous to have as many answers as possible, each yielding some insight and information.

3) This theorem may be easily generalized to systems governed by the more general equation $[A + C(t)]\boldsymbol{x}'(t) = [B + D)(t)]\boldsymbol{x}(t) + \boldsymbol{u}(t)$.

Systems of the form $[A + \varepsilon C(t)]x' = [B + \varepsilon D(t)]x + u(t)$, with A and B constant matrices, small ε, and $C(t)$ and $D(t)$ (periodic matrices of period T), are discussed next. Equations of this kind occur when the parameters of a system governed by $Ax' = Bx$ are subjected to small periodic variations. The perturbations may occur because of uncertainties in modeling, disturbances, or both.

Corollary. Let B denote a constant $(n \times n)$ matrix and c_1 a positive real number, such that all the eigenvalues of $(B + c_1 I)$ have negative real parts. Let $C(t)$ and $D(t)$ denote $(n \times n)$ periodic matrix valued functions of t, with measurable elements. The elements of C are assumed to be differentiable. Let $\boldsymbol{x}$ be a complex n-vector valued differentiable function of t on $[0,\infty)$, such that

$$[I + \varepsilon C(t)]\boldsymbol{x}'(t) = [B + \varepsilon D(t)]\boldsymbol{x}(t) + \boldsymbol{u}(t) \tag{32}$$

with $e^{c_1 t}u(t) \in L_2^n[0,\infty)$ for $c_1 > 0$ and $\varepsilon \ll 1$.

If $(\xi_j + i\eta_j)$, $j = 1,...,n$ are the eigenvalues of B, then let

$$\xi \triangleq \{\xi_j: |\xi_j = c_1| \text{ is minimum over } j = 1,...,n\}$$

If $\|C(t)\| \le c$, and $|\xi - c_1| > 0(\varepsilon)$, then $x \in L_\infty^n[0,\infty)$ and there exist positive constants c_2, c_3, c_4, such that

$$\|x(t)\| \le 1/(1-\varepsilon c)[c_2 e^{-c_1 t} + c_3 + c_4 \int_0^t \|u(\tau)\|\,\mathrm{d}\tau] \text{ for } t \ge 0 \tag{33}$$

Note that $\xi > 0(\varepsilon)$ if $\xi = 0(\varepsilon^n)$ where $n > 1$.

Proof. The elements of $C(t)$ and $D(t)$, being periodic, are uniformly bounded. The theorem is applicable and Eq. (32) is

L_∞ stable if $\|\varepsilon C(t)\| \le \varepsilon c < 1$, which is true, and

$$\varepsilon c + 1/|\xi - c_1| \sup_{t \ge 0} \Lambda\{D(t) + C'(t) - BC(t)\} < 1 \qquad (34)$$

which is true if $|\xi - c_1| > 0(\varepsilon)$.

Remark. This corollary, too, may be generalized to systems governed by the more general equation

$$[A + \varepsilon C(t)]x'(t) = [B + \varepsilon D(t)]x(t) + u(t) \qquad (35)$$

This section now closes with two proposed extensions to this work:

1) Determination of a closed-form expression for $\mathscr{L}^{-1}\{(s^2A + sB + C)^{-1}\}$ would prove extremely useful, as it would enable the second-order system to be analyzed as such.

2) The analysis was done only for systems with the stable constant parts. Stable systems with unstable constant parts form a class not yet explored.

Conclusion

The authors have been recently engaged in the study of the stability of time-varying systems. In continuation with the earlier work, L_∞ stability is studied in this Note. A lemma dealing with the L_∞ stability of an integral equation, resulting from the differential equation of the system under consideration, is first proved. Using it, the main result on L_∞ stability is derived, according to which the system is L_∞ stable, if the eigenvalues of the coefficient matrices are related in a simple way. A corollary of the theorem deals with constant coefficient systems perturbed by small periodic terms, a problem of great importance in its own right. Although the mathematical analysis may seem a little abstract, and may at first deter a practicing engineer not trained in the methods of modern mathematics from using them, the final results are easy to verify even for complex systems.

References

[1]Pradeep, S. and Shrivastava, S. K., "Stability of Dynamical Systems: An Overview," *Journal of Guidance, Control, and Dynamics* (to be published).

[2]Shrivastava, S. K., "Stability Theorems for Multidimensional Linear Systems with Variable Parameters," *ASME Journal of Applied Mechanics*, Vol. 48, March 1981, pp. 174–176.

[3]Shrivastava, S. K. and Pradeep, S., "On the Stability of Multidimensional Linear Time Varying Systems," *Journal of Guidance, Control and Dynamics*, Vol. 8, Sept.–Oct. 1985, pp. 579–583; also, AIAA Paper 84-1954, Aug. 1984.

[4]Pradeep, S. and Shrivastava, S. K., "On the Asymptotic Behavior and Boundedness of Systems with Time Varying Coefficients," *Acta Astronautica* (submitted for publication).

[5]Pradeep, S. and Shrivastava, S. K., "On the L_2 Stability of Multidimensional Linear Time Varying Systems," *Journal of Astronautical Sciences* (to be published).

[6]Sandberg, I. W., "On the Solutions of Systems of Second-Order Differential Equations with Variable Coefficients," SIAM *Journal on Control*, Ser. A, Vol. 2, No. 2, 1965, pp. 192–198.

Nutation Damping Using a Pivotable Momentum Wheel

C. Hubert* and D. Bruno†
General Electric Company, Princeton, New Jersey

Introduction

A PIVOTABLE momentum wheel can be used as an effective active nutation damping device on momentum-biased spacecraft. Previously described pivot-based active nutation control systems provide linear, essentially viscous damping.[1–3] Although linear damping is effective, it can be slow, requiring many nutation cycles to damp a large initial nutation. The current control system provides much faster damping by driving the pivot with a stepper motor controlled by simple nonlinear digital logic. Simulations and flight data show that nutation angles of 1 deg or more can be damped within two nutation cycles.

System Description

In the following discussion, the spacecraft's yaw, roll, and pitch axes are denoted as X, Y, and Z, respectively. The momentum wheel's spin axis is nominally aligned with the pitch axis. The wheel speed is biased to maintain gyroscopic stiffness and modulated to control pitch. A single-axis pivot mechanism can rotate the wheel in either direction about the roll axis.

Figure 1 outlines the Pivot-Actuated Nutation Damper (PANDA) digital logic. The PANDA pivots the wheel about the roll axis in response to a roll-rate reference. This rate may be obtained directly from a gyro or derived indirectly from an external reference, such as an Earth sensor. In either case, the rate signal is processed by a bandpass filter that passes nutation frequency while removing any biases, structural frequencies, and noise. The filtered roll-rate signal is monitored until it exceeds a predetermined threshold. When the threshold is crossed, the logic determines the sign at the crossing and then determines the amplitude of the sinusoidal rate signal. This amplitude is proportional to the nutation angle. When the filtered roll rate next crosses zero, the momentum wheel pivots through an angle proportional to the roll-rate amplitude. If the zero crossing is from negative to positive, the wheel pivots about the positive roll axis. If the crossing is from positive to negative, the wheel pivots about the negative roll axis. (These signs assume that the spacecraft is dual-spin stable. If the system is dynamically unstable, the sign of the pivoting should be reversed.) One-half nutation period after the first pivoting begins, the wheel pivots an equal angle in the opposite direction. Thus, at the end of the nutation damping cycle, the wheel returns to its initial position. Once the cycle completes, the PANDA logic resumes monitoring the roll rate. For greatest effectiveness, the wheel pivoting time should be short compared to the nutation period. Simulations suggest that a good upper limit for the wheel pivoting time (in one direction) is between 5 and 10% of the nutation period.

Figure 2 illustrates the PANDA dynamics. This momentum diagram shows the trajectory of the tip of the spacecraft's angular momentum vector projected on the body-fixed yaw-roll (X-Y) plane. For simplicity, the figure assumes that the spacecraft's yaw and roll inertias are equal.

In Fig. 2, the momentum wheel is nominally aligned with the positive pitch (Z) axis. If there is no nutation, the nominal trajectory is simply equilibrium point a. When the spacecraft nutates, however, the momentum vector follows a circular

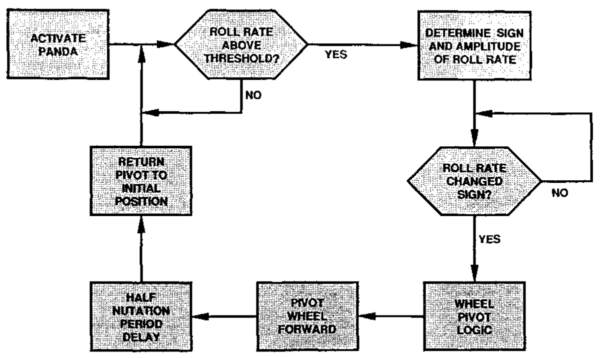

Fig. 1 Simplified pivot-actuated nutation damper logic.

Received June 13, 1988.

*Manager, Control Analysis, Astro-Space Division. Associate Fellow AIAA.

†Principal Member, Technical Staff, Astro-Space Division. Senior Member AIAA.